Question	Scheme	Marks	AOs
1(a)	$\mathrm{f}(-3)=2(-3)^3+5(-3)^2+2(-3)+15$ $=-54+45-6+15$	M1	1.1b
	$\mathrm{f}(-3)=0 \Rightarrow (x+3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a=2,\ b=-1,\ c=5$	M1	1.1b
	All of: $a=2,\ b=-1,\ c=5$	A1	1.1b
		(2)	
(c)	$b^2-4ac=(-1)^2-4(2)(5)$	M1	2.1
	$b^2-4ac=-39$ which is <0 so the quadratic has no real roots so f(x) = 0 has only 1 real root	A1	2.4
		(2)	
(d)	$(x=)\ 2$	B1	2.2a
		(1)	
			(7 marks)

Notes

(a)

M1: Attempts f(−3). Attempted division by $(x+3)$ or $\mathrm{f}(3)$ is M0
Look for evidence of embedded values or two correct terms of
$\mathrm{f}(-3)=-54+45-6+15=\ldots$

A1: Achieves and states f(−3) = 0, and makes a suitable conclusion. Sight of f(x)=0 when $x=-3$ is also acceptable.
It must follow M1. Accept, for example, $\mathrm{f}(-3)=0 \Rightarrow (x+3)$ is a factor

This may be seen in a preamble before finding $\mathrm{f}(-3)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Correct method implied by values for at least 2 correct constants. Allow embedded in their $\mathrm{f}(x)$ or within their working if they use algebraic division/other methods which may be seen in part (a) and used in part (b).

A1: All values correct. Allow embedded in their $\mathrm{f}(x)$ or seen as the quotient from algebraic division. Isw incorrectly stated values of a b and c following a correct quadratic expression seen.

$$\begin{array}{r} 2x^2-x+5 \\ x+3\overline{\smash{\big)}\,2x^3+5x^2+2x+15} \\ \underline{2x^3+6x^2} \\ -x^2+2x \\ \underline{-x^2-3x} \\ 5x+15 \\ \underline{5x+15} \\ 0 \end{array}$$

scores M1A1

(c)

M1: Either:

- considers the discriminant using their *a*, *b* and *c* (does not need to be evaluated) $(b^2-4ac=)\ (-1)^2-4(2)(5)$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2). Discriminant $=-39$ is sufficient for M1
- attempts to complete the square so score for $2\left(x\pm\frac{1}{4}\right)^2+...$
- attempts to find the roots of the quadratic using the formula. The values embedded in the formula score this mark. $\dfrac{1\pm\sqrt{(-1)^2-4\times2\times5}}{2\times2}$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2)
- Sketches a graph of the quadratic. It must be a U shaped quadratic which does not cross the *x*-axis.

A1: Provides a correct explanation from correct working. They must

- Have a correct calculation
- Explanation that the quadratic has no (real) roots
- Minimal conclusion stating that $f(x)=0$ has only one root

eg $b^2-4ac=-39<0$ so only one root is M1A0 (needs to explain the quadratic has no real roots)

eg $2\left(x-\frac{1}{4}\right)^2+\frac{39}{8}>0$ **so no real roots** (for the quadratic) **so** (f(*x*) has) **only one (**real**) root** is M1A1

The value of the discriminant, completed square form $2\left(x-\frac{1}{4}\right)^2+\frac{39}{8}$ or roots of the quadratic $\left(=\frac{1\pm\sqrt{39}i}{4}\right)$ must be correct.

If they sketch the quadratic graph it must be a U shaped quadratic which crosses the *y*-axis at 5 and has a minimum in the 1st quadrant. They must explain that the graph does not cross the *x*-axis so no real roots for the quadratic so only one root for $f(x)=0$.

(d)

B1: 2 condone (2, 0)

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Question	Scheme	Marks	AOs
2	$\mathrm{f}(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = ...$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

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Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a = 3$, $\mathrm{f}(x) = 3x^3 + 10x^2 - 9x - 4$ and shows that $(x-1)$ is a factor of this $\mathrm{f}(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3 + 10x^2 - 9x - 4 = (x-1)\left(3x^2 + 13x + 4\right)$ Hence $a = 3$

E.g. 2: $\mathrm{f}(x) = 3x^3 + 10x^2 - 9x - 4$, $\mathrm{f}(1) = 3 + 10 - 9 - 4 = 0$ Hence $a = 3$

The solutions via this method must end with the value for a to score the A1

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Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

Alt (1) by inspection which may be seen in a table/g

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$$ and sets terms in x equal

$$-3a=-(10+a)+4\Rightarrow 2a=6\Rightarrow a=3$$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

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Alt (2) By division:

$$\begin{array}{r l}
 & ax^2+(\pm10\pm a)x+(10-2a) \\
x-1 \big) & ax^3+10x^2\quad-3ax-4 \\
 & \underline{ax^3-ax^2} \\
 & \quad(10+a)x^2-3ax \\
 & \quad\underline{(10+a)x^2-(10+a)x} \\
 & \qquad\qquad(-2a+10)x
\end{array}$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

Question	Scheme	Marks	AOs
3	Sets $f(-2)=0 \Rightarrow (-2-4)\,((-2)^2-3\times-2+k)-42=0$	M1	3.1a
	$-6\,(k+10)=42 \Rightarrow k=...$	M1	1.1b
	$k=-17$	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Attempts $f(-2)=0$ leading to an equation in *k*. So $(-2-4)\,((-2)^2-3\times-2+k)-42=0$ is fine

Condone slips but expect to see a first bracket of $(-2-4)$.

$"-42"$ must not be omitted but could appear as +42 with a sign slip.

There may have been attempts to expand $f(x)=(x-4)(x^2-3x+k)-42$ before attempting to set $f(-2)=0$. This is acceptable and condone slips/errors in the expansion, but the 42 must be present. FYI the expanded (and simplified) $f(x)=x^3-7x^2+(12+k)x-4k-42$

M1: Solves a **linear** equation in *k* as a result of setting $f(\pm2)=0$.

The ±42 must be there at some point when the substitution is made.

Allow minimal evidence here. A linear equation leading to a solution is fine.

If f (*x*) is expanded then it is dependent upon being a cubic which contains a *kx* term and a ‘42’

A1: $k=-17$ correct answer following correct work but allow recovery from invisible brackets

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Answers of $k=-17$ may appear with very little or no working, perhaps via trial and improvement. If so, then marks can only be allocated if evidence is shown.

E.g. $k=-17 \Rightarrow f(x)=(x-4)\,(x^2-3x-17)-42$

$f(-2)=(-6)\times(-7)-42=0$. Hence $(x+2)$ is a factor.

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More difficult alternative methods may be seen

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Alt I : You may see attempts via division / inspection

$$\begin{array}{r} x^2-9x+(k+30) \\ x+2\overline{\big)x^3-7x^2+(12+k)x-4k-42} \\ \\ \\ \hline -6k-102 \\ \hline \end{array}$$

Then sets remainder $-6k-102=0 \Rightarrow k=-17$

M1: For dividing their cubic by (*x*+2) which has both an *x* and a constant coefficient in *k*, leading to a quadratic quotient and a linear remainder in *k* which is then set = 0

M1: Solves a equation resulting from setting a linear remainder in *k* equal to 0 . It is dependent on the first M via this route

A1: Completely correct with $k=-17$

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Alt II: You may also see a grid or an attempt at factorisation via inspection

	x^2	$-9x$	$-2k-21$
x	x^3	$-9x^2$	$(-2k-21)x$
$+2$	$2x^2$	$-18x$	$-4k-42$

OR $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)\left(x^2-9x-2k-21\right)$

which should be followed by equating the *x* terms to form an equation in k

$$12+k=-18-2k-21 \Rightarrow 3k=-51 \Rightarrow k=-17$$

OR $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)\left(x^2-9x+k+30\right)$

which should be followed by equating the constant terms to form an equation in k

$$-4k-42=2\left(k+30\right) \Rightarrow 6k=-102 \Rightarrow k=-17$$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt at factorising E.g. $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)\left(x^2+bx+c\right)$ and attempting to set up three equations in *b*, *c* and *k*. E.g $2+b=-7, 2b+c=12+k, 2c=-4k-42$

The expanded f(*x*) must be a cubic which contains both a *kx* term and a '42'

M1: Solves the equations set up from an allowable equation to find *k*. It is dependent via this route.

A1: Completely correct with $k=-17$

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Question	**Scheme**	**Marks**	**AOs**
4 (a)	Substitutes $x=\frac{1}{2}$ into $y=2x^3+10$ **and** $y=42x-15x^2-7$ and finds the *y* values for both	M1	1.1b
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*	2.4
		(2)	
(b)	Sets $42x-15x^2-7=2x^3+10 \Rightarrow 2x^3+15x^2-42x+17=0$	M1	1.1b
	Deduces that $(2x-1)$ is a factor and attempts to divide	dM1	2.1
	$2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$	A1	1.1b
	Solves their $x^2+8x-17=0$ using suitable method	M1	1.1b
	Deduces $x=-4+\sqrt{33}$ (see note)	A1	2.2a
		(5)	
			(7 marks)
Notes:			

(a)

M1: Substitutes $x=\frac{1}{2}$ into both $y=2x^3+10$ and $y=42x-15x^2-7$ and finds *y* values

Sight of just the *y* values at each is sufficient for this mark only.

Alternative: Sets $42x-15x^2-7=2x^3+10 \Rightarrow$ cubic and substitutes $x=\frac{1}{2}$ into the expression, attempts $f\left(\frac{1}{2}\right)$ or else attempts to divide the cubic = 0 by $(2x-1)$ or $\left(x-\frac{1}{2}\right)$. Condone $f\left(\frac{1}{2}\right)=0$ without calculations for this mark only.

A1*: Correct calculations must be seen with a minimal conclusion that curves intersect (at $x=\frac{1}{2}$).

E.g. $2\left(\frac{1}{2}\right)^3+10=10.25$ and $42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7=10.25$ so curves intersect.

Acceptable alternatives are:

$f(x)=42x-15x^2-7-2x^3-10, f\left(\frac{1}{2}\right)=42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7-2\left(\frac{1}{2}\right)^3-10=0 \Rightarrow$ so curves intersect

$f(x)=2x^3+15x^2-42x+17 \Rightarrow \left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$ so $x=\frac{1}{2}$ is a root so curves intersect

$f(x)=2x^3+15x^2-42x+17 \Rightarrow (2x-1)\left(x^2+8x-17\right)$ so $(2x-1)$ is a factor hence curves intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

Special case: Scores M1 A0 with or without a conclusion

This is presumably done using a calculator and requires all three roots exact or correct to 3sf

$f(x)=2x^3+15x^2-42x+17=0$

$\Rightarrow x=0.5, 1.74, -9.74$

(b) **This part requires candidates to show all stages of their working.**
Answers without working will not score any marks
A method must be seen which could be from part (a) which must then be continued in (b)

M1: Sets $42x-15x^2-7=2x^3+10$ and proceeds to 4 term cubic equation.
Condone slips, e.g. signs. Terms do not have to be on one side of the equation.

dM1: For the key step in attempting to "divide" the cubic by $(2x-1)$

If attempted via inspection look for correct first and last terms

E.g. $2x^3+15x^2-42x+17=(2x-1)\left(x^2+...\pm17\right)$ if cubic expression is correct

If attempted via division look for correct first and second terms

$$2x-1\overset{\displaystyle x^2+8x}{\overline{\big)\,2x^3+15x^2-42x+17}}$$ if cubic expression is correct

It is acceptable for an attempt to divide by $\left(x-\frac{1}{2}\right)$. It is easily marked using the same guidelines, e.g. $2x^3+15x^2-42x+17=\left(x-\frac{1}{2}\right)\left(2x^2+16x...\right)$

A1: $2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$ o.e. $\left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$

This may be implied by sight of $\left(x^2+8x-17\right)$ or $\left(2x^2+16x-34\right)$ in a "division" sum.

M1: Solves their quadratic $x^2+8x-17=0$ using a suitable method including calculator. You may need to check this. It is not completely dependent upon the previous M's but an attempt at a full method must have been seen. So look for

- the two equations being set equal to each other and some attempt made to combine
- some attempt to "divide" the result by $(2x-1)$ o.e. allowing for flaws in the method

A1: Gives $x=-4+\sqrt{33}$ o.e. only. The $x=-4-\sqrt{33}$ must not be included in the final answer.

Allow exact unsimplified equivalents such as $x=\frac{-8+\sqrt{132}}{2}$. ISW for instance if they then put this in decimal form.

Question	Scheme	Marks	AOs
5(a)	$\left(\mathrm{f}(a)=\right)\ 4a^3+5a^2-10a+4a=0 \Rightarrow a(....)=0$	M1	3.1a
	$a(4a^2+5a-6)=0$ *	A1*	1.1b
		(2)	
(b)(i)	$a=\dfrac{3}{4}$	B1	2.2a
(ii)	$4x^3+5x^2-10x+4\times\text{"}\dfrac{3}{4}\text{"}=3 \Rightarrow 4x^3+5x^2-10x\ (=0)$	M1	1.1b
	$x=0$	B1	1.1b
	$x=\dfrac{-5\pm\sqrt{185}}{8}$	A1	1.1b
		(4)	
			(6 marks)

Notes

(a)

M1: Attempts $\mathrm{f}(a)=0$ leading to an equation in *a* only and attempts to take a factor of *a* out. Condone slips. The = 0 may appear on a later line which is fine, but must be seen at some point in their solution in part (a).

A1*: Achieves the given answer with no errors including brackets.
Minimum acceptable is $4a^3+5a^2-10a+4a=0 \Rightarrow a(4a^2+5a-6)=0$
If the = 0 is absent at the start of their solution, it must appear before achieving the given answer.
Do not allow attempts to find the value of *a* and substitute that into $\mathrm{f}(x)$

More difficult alternative methods may be seen:

Alt 1: You may see attempts via division / inspection

$$
\begin{array}{rllll}
 & 4x^2 & +(5+4a)x & & +(-10+5a+4a^2) \\
\cline{2-5}
x-a \big) & 4x^3 & +5x^2 & -10x & +4a \\
 & 4x^3 & -4ax^2 & & \\
 & & (5+4a)x^2 & -10x & \\
 & & (5+4a)x^2 & -a(5+4a)x & \\
 & & & (-10+5a+4a^2)x & +4a \\
 & & & (-10+5a+4a^2)x & -a(-10+5a+4a^2) \\
 & & & & -6a+5a^2+4a^3
\end{array}
$$

Then sets remainder $-6a+5a^2+4a^3=0$

M1: For dividing the cubic by $(x-a)$ leading to a quadratic quotient in *x* and a cubic remainder in *a* which is then set = 0 and attempts to take a factor of *a* out.

A1*: Completely correct with $a(4a^2+5a-6)=0$

Alt 2: You may also see a grid or an attempt at factorisation via inspection

	$4x^2$	$+(5+4a)x$	$+(-10+5a+4a^2)$
x	$4x^3$	$+(5+4a)x^2$	$(-10+5a+4a^2)x$
$-a$	$-4ax^2$	$-a(5+4a)x$	$-a(-10+5a+4a^2)$

OR $4x^3+5x^2-10x+4a \equiv (x-a)(4x^2+px-4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously.

$-10=-ap-4$ and $5=-4a+p \Rightarrow p=5+4a$

$\Rightarrow -10=-a(5+4a)-4 \Rightarrow 4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2. Condone slips.

A1*: $4a^2+5a-6=0 \Rightarrow a(4a^2+5a-6)=0$ Completely correct with no errors.

(b) Mark (i) and (ii) together

(i)

B1: Deduces that $a=\frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected

(ii)

M1: Attempts to substitute their $a=\frac{3}{4}$ (which must be positive) into $\text{f}(x)$, sets their $\text{f}(x)=3$ and collects terms on one side (= 0 may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s).

B1: $(x=)\ 0$

A1: $(x=)\ \frac{-5\pm\sqrt{185}}{8}$ (**and these values only**) or exact equivalent (ignore 0 for this mark). Withhold this mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3+5x^2-10x$ without taking a factor of or dividing by x first to **see the quadratic factor**. Isw once the correct answers are seen if they proceed to provide rounded answers after.

e.g. $4x^3+5x^2-10x+4\times"\frac{3}{4}"=3 \Rightarrow 0,\ \frac{-5\pm\sqrt{185}}{8}$ is M0B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 0,\ \frac{-5\pm\sqrt{185}}{8}$ is M1B1A0

e.g. $4x^3+5x^2-10x=0 \Rightarrow 4x^2+5x-10=0 \Rightarrow \frac{-5\pm\sqrt{185}}{8}$ is M1B0A1

e.g. $4x^3+5x^2-10x=0 \Rightarrow x(4x^2+5x-10)=0 \Rightarrow 0,\ \frac{-5\pm\sqrt{185}}{8}$ is M1B1A1